Mastery of the counting principles in toddlers: A crucial step in the development of budding arithmetic abilities?

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A R T I C L E   I N F O

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A B S T R A C T

Counting abilities have been described as determinative precursors for a good development of later arithmetic abilities. Mastery of the stable order, the one–one-correspondence and the cardinality principles can be seen as essential features for the development of counting abilities. Mastery of the counting principles in kindergarten was assessed in a large group of children with a broad range of arithmetic abilities (N = 423). Not all children mastered the counting principles by the end of kindergarten. Mastery of the counting principles in kindergarten was predictive for arithmetic abilities one year later in first grade, especially for scores on arithmetic achievement tests. Children sharing a common educational background tend to have more similar scores on arithmetic tests, yet the importance of mastery of the essential counting principles in the prediction of later arithmetic achievement was the same for all classrooms.

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1. Introduction

While arithmetic abilities mainly improve during the early school years, it is clear that this developmental process begins before formal schooling starts (Resnick, 1989; Sophian, Wood, & Vong, 1995). This study focused on counting abilities as one of the main early arithmetic abilities in toddlers. Geary (2004) stated that children's counting abilities could be seen as a combination of inherent constraints and inductions based on counting experiences. Those early inherent constraints had been described by Gelman and Galistel (1978) as the five conceptual principles in counting. The stable-order principle implies that the order of number words must be invariant across counted sets. The one–one-correspondence principle holds that every number word can only be attributed to one counted object. Once the cardinality principle is acquired, children know that the value of the last number word represents the quantity of the counted objects. The abstraction principle holds that objects of every kind can be counted. The last counting principle is the order-irrelevance principle. This holds that the objects in a set can be counted in any sequence without influencing the counting result (Gelman & Galistel, 1978).

Researchers agree on the principles as described by Gelman and Gallistel (Dehaene, 1992) but a lot of discussion exists on the role of those principles in the explanation of the development of arithmetic abilities (Sophian, 1997). Gelman and Galistel (1978) and Gelman (1990) argued that there are three innate counting principles that guide children's acquisition of the counting list: the stable-order principle, the one–one-correspondence principle and the cardinality principle. Mastery of these three principles forms the skeletal structure for children's emerging knowledge of counting (Geary, 2004; Gelman & Meck, 1983). The assumption that mastery of these three counting principles is an innate ability is debated heavily, but researchers agreed on the importance of this id (Wynn, 1992). Moreover mastery of the cardinality principle can be seen as one of the main foundations for a good development of several arithmetic abilities (Dowker, 2005). The other two principles, the abstraction principle and the order-irrelevance principle have been approached as the unessential counting abilities. Ignoring the abstraction principle does not lead to mistakes in the counting process on itself and order irrelevance is an inessential feature of counting, because violation of that principle does not result in incorrect counting (Briars & Siegler, 1984; Le Fevre et al., 2006).

During the development of counting abilities, children incorporate both essential and unessential principles in their counting knowledge but do not keep using the unessential ones over time. Le Fevre et al. (2006) found that children with good arithmetic abilities went faster through this developmental learning process than children who performed worse on arithmetic abilities tests.

Children do not reach mastery of the three essential features at the same time. Knowledge of the stable-order principle is reliably first of all, followed by the one–one-correspondence principle, while mastery of the cardinality principle was found to develop the slowest (Butterworth, 2004; Fuson, 1988). The development of mastery of the stable-order principle was situated quite early. Le Fevre et al. (2006) found that knowledge of this principle was very good for children in kindergarten. Children in first grade even reached an accuracy level equal to that of adults. Almost the same indication of age for full understanding of the one–one-correspondence principle...
was found. Children had a rather good understanding of the one–one-correspondence by the age of five (Briars & Siegler, 1984) and this understanding improved as grade and counting abilities improved (Le Fèvre et al., 2006). Wynn (1992) situated the comprehension of these abilities a lot earlier. She found that children of two to three years old could already assign one number word to one referent, indicating that children already mastered this principle at that age. Yet Wynn (1992) indicated that most evidence was based on cross-sectional studies and that a lot of individual variation existed, making well-founded judgements difficult. There is at least as much debate concerning the mastery of the cardinality principle. Gelman and Meck (1983) found the principle to be mastered around the age of three and Wynn (1992) argued that this understanding just began at three and a half. On the contrary, Freeman, Antonucci, and Lewis (2000) found that children could determine quantities not until the age of four and a half, and found that the first signs of a principled understanding of cardinality were not apparent before the age of five.

1.1. Objectives and research questions

The goal of the current research was to examine the mastery of the essential counting principles in toddlers and its relation with arithmetic abilities in first grade. It was expected that most children mastered the different counting principles by the end of kindergarten (our first hypothesis). It was further supposed that the better children mastered the counting principles in the last kindergarten class, the better they scored on arithmetic tests in first grade (our second hypothesis).

2. Method

2.1. Participants

This study has been carried out in a total group of 200 boys and 223 girls. All children were Caucasian native Dutch-speaking children living in the Flanders. A socio-economic status was derived from the total number years of scholarship of the parents (starting from the beginning of elementary school), with a mean of 14.88 years ($SD = 2.45$ years) for mothers and 14.65 years ($SD = 2.95$ years) for fathers.

The children were tested in the second part of the school year (April or May) in the last kindergarten class and again in the same period when they were in first grade. The children had a mean age of 70.02 months ($SD = 4.01$ months) and attended on average 7.42 months ($SD = 1.03$ months) of school in the last kindergarten class when tested the first time. Children who received professional treatment for learning or other disabilities ($n = 59$) were excluded out of the sample.

2.2. Materials

All children were tested in kindergarten on their knowledge of counting principles. Follow-up assessment with two arithmetic tests was conducted in first grade.

2.2.1. Assessment of the mastery of the counting principles in kindergarten

All counting abilities were tested with different items of the TEDI-MATH (Grégoire, Noel, & Van Nieuwenhoven, 2004). The TEDI-MATH has proven to be a well validated (Desoete, 2006, 2007a,b) and reliable instrument, values for Cronbach’s Alpha for the different subtests vary between .70 and .97 (Desoete & Grégoire, 2007; Grégoire et al., 2004). Mastery of the stable-order principle was assessed using accuracy in counting numbers. Children had to count till 30. Children earned two points when they could count faultless in the first attempt; one point was earned when two attempts were necessary. Children were further asked to count forward with an upper bound (one point) and with a lower bound (one point). Children who gained three or more points on these items were classified as mastering the stable-order principle.

Mastery of the one–one-correspondence principle was assessed using accuracy in counting linear and random patterns of drawings. Children were asked to count for example all the rabbits on a drawing. Four items were presented. One point was given for every correct answer. Children who gained three or more points on these items were classified as mastering the one–one-correspondence principle.

Mastery of the cardinality principle was assessed with a ‘how many’-task (Wynn, 1990, 1992). This kind of task elicits responding with the last number word of the counting sequence and fits well the cardinality principle as postulated by Gelman and Gallistel (1978; Bermejo, Morales, & deOsuna, 2004). After presenting counting items, children were asked ‘How many are there in total?”. Four items were presented. One point was given for every correct answer. Children who gained three or more points on these items were classified as mastering the cardinality principle.

The Cronbach’s Alpha for the three counting measures together was .78.

2.2.2. Arithmetic tests in first grade

Two arithmetic tests were used. The Kortrijk Arithmetic Test Revision (Kortrijkse Rekentest Revision, KRT-R; Baudonck et al., 2006) is a standardized test on arithmetical achievement which requires that children solve 30 mental arithmetics (e.g., $4 + 1 = ...$) and 30 number knowledge (e.g., $1$ more than $3$ is $...$) tasks. For the current analyses, total percentile scores based on national norms were used. The psychometric value of the KRT-R has been demonstrated on a sample of 3246 children. A validity coefficient (correlation with school results) and reliability coefficient (Cronbach’s Alpha) of .50 and .92 respectively were found for first grade.

The TTR (De Vos, 1992) is a test consisting of 80 arithmetic number fact problems on addition and subtraction. The total number of correct items was used as score for the analyses. The psychometric value of the TTR has been demonstrated on a sample of 10,059 children in total.

2.3. Procedure

The children were recruited in 25 randomly selected schools, 9 schools were located in a city while 16 of them were located rural. All parents received a letter with the explanation of the research and could submit informed consent in order to participate. Children were tested during school time in a separate and quiet room. Toddlers were tested individually. The total duration of the individual testing was about 40 min. In first grade the arithmetic tests were assessed classically. During the follow-up testing, we had a drop-out of 12.53%, 370 children completed all the tests and were included for the analyses. The test leaders all received training in the assessment and interpretation of the tests.

3. Results

Table 1 gives an overview of the descriptives for the scores on the counting items and for the arithmetic abilities as measured in first grade.

<table>
<thead>
<tr>
<th>Kindergarten</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting abilities</td>
<td>9.25</td>
<td>2.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>First grade</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical facility</td>
<td>18.12</td>
<td>7.27</td>
</tr>
<tr>
<td>Arithmetical achievement</td>
<td>64.15</td>
<td>29.82</td>
</tr>
</tbody>
</table>

Note. Total raw scores on counting items were used.
3.1. Hypothesis one: mastery of counting abilities in kindergarten

In this sample 62.4% of children mastered the stable-order principle, 95.3% mastered the one–one-correspondence principle and 65.7% mastered the cardinality principle. Only 44.2% of children mastered the three counting principles by the end of kindergarten. Table 2 gives an overview of the number and proportion of toddlers who mastered the different principles.

3.2. Hypothesis two: prediction of arithmetic abilities

Since the children in this study are clustered in classrooms and thus not sampled randomly and independently, the multilevel structure of the data was first investigated by calculating the intraclass correlations for both dependent variables. The intraclass correlation is calculated as the proportion of the between-group variance relative to the sum of the between- and within-group variance. It corresponds to a null model with no explanatory variables. The intraclass correlation for arithmetical achievement was .50, and for numerical facility it was .40 (see also Table 3).

These indices indicate that between 40 and 50% of the variance in the dependent variables could be explained by the cluster structure of the data, meaning that going to the same school tends to exert an influence on scores on arithmetic tests.

Multilevel analyses were performed with counting skills as the independent predictor for arithmetic achievement and numerical facility respectively. A total score for counting was calculated by adding the raw scores on the items for the three counting principles in kindergarten. The range of this score varied from zero to twelve and was then normalized enabling an easier interpretation. No school variables were added to the model. The scores on the arithmetic tests were treated at Level 1 and classrooms were classified as Level 2. The results of the analyses are presented in Table 4.

In the fixed part of the model we see that children with better counting skills in kindergarten tended to perform better on arithmetic tests in first grade. The individual level intercept variance is .56 for arithmetic achievement and .63 for numerical facility. The classroom level intercept variance is .57 for arithmetic achievement and .42 for numerical facility.

The random part of the model shows that classrooms differ in their mean performances. Yet no significant slope variance was found for the scores on the arithmetic tests between the different classrooms (see also Table 4).

4. Discussion

The goal of the current research was to have more insight in the importance of mastering the essential counting principles in kindergarten. The results showed that almost all children had a thorough command of the one–one-correspondence principle. In contrast to the existing literature (Briars & Siegler, 1984; Freeman et al., 2000; Gelman & Meck, 1983; Le Fevre et al., 2006; Wynn, 1992) the results pointed out that not all children in the dataset mastered the other two essential counting principles by the end of kindergarten. Concerning the stable-order principle it was found that about 60% of children mastered this principle by the end of the last kindergarten class. Finally about two thirds of the children in the last kindergarten class could apply the cardinality principle when counting. This finding is in contrast to the findings that the understanding of the cardinality principle is the most difficult principle, developing relatively late (Butterworth, 2004; Fuson, 1988). The results showed that almost one third of children still had problems with understanding the stable order or the cardinality principle. More than half of children did not master the three counting principles by the end of kindergarten.

The second hypothesis could be confirmed. The better children performed on the counting items in the last kindergarten class, the better they performed on the two arithmetic tests in first grade. Based on the counting scores of children at the end of the last kindergarten class, 14% of the variance in the scores for arithmetic achievement in first grade could be explained. These results confirm the role of counting abilities in the development of proficient arithmetic strategies (Blöte, Lieferring, & Ouwehand, 2006; Le Fevre et al., 2006; Stock, Desoete, & Roeyers, 2007). In addition significant proportions of the variance in the scores for the number fact knowledge test in first grade could be explained, but this proportion was limited to only 5%. Although earlier research also stressed the role of counting abilities in the automatisation of arithmetic facts (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Van de Rijt & Van Luit, 1999), it was found that mastery of the essential counting principles was far more important for the command of domain specific arithmetic abilities than for the automatisation of arithmetic facts. A possible explanation for this difference could be that the current study has taken into account classroom variances and that this was responsible for the larger role of counting in the prediction of numerical facility in former studies. This difference could also partly be explained by the fact that the counting items were more related to the items of the arithmetic achievement test than to the items of the numerical facility test. Moreover, the results pointed out that a large part of the variance in arithmetic achievement in first grade can be associated with differences between schools.

Table 2
Percentages for mastering the stable-order principle, the one–one-correspondence or cardinality principle.

<table>
<thead>
<tr>
<th>Principle</th>
<th>Non mastery</th>
<th>Inconsistent</th>
<th>Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable-order principle (SOP)</td>
<td>37.6%</td>
<td>/</td>
<td>62.4%</td>
</tr>
<tr>
<td>One–one-correspondence principle (OOP)</td>
<td>4.7%</td>
<td>/</td>
<td>95.3%</td>
</tr>
<tr>
<td>Cardinality principle (CP)</td>
<td>34.3%</td>
<td>/</td>
<td>65.7%</td>
</tr>
<tr>
<td>SOP + OOP</td>
<td>4.2%</td>
<td>36.7%</td>
<td>59.1%</td>
</tr>
<tr>
<td>SOP + CP</td>
<td>16.5%</td>
<td>40.2%</td>
<td>43.3%</td>
</tr>
<tr>
<td>OOP + CP</td>
<td>4%</td>
<td>30.3%</td>
<td>65.7%</td>
</tr>
<tr>
<td>All 3 principles</td>
<td>55.8%</td>
<td>/</td>
<td>44.2%</td>
</tr>
</tbody>
</table>

Note. The chosen cut-off for the mastery of the counting principles is arbitrary.

Table 4
Mixed model analyses of arithmetical achievement and numerical facility scores with counting skills.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Arithmetical achievement</th>
<th>Numerical facility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
</tr>
<tr>
<td>Fixed intercept</td>
<td>−.13</td>
<td>.13</td>
</tr>
<tr>
<td>Counting skills</td>
<td>.33*</td>
<td>.06</td>
</tr>
<tr>
<td>Random</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 2 intercept</td>
<td>.41*</td>
<td>.13</td>
</tr>
<tr>
<td>Level 2 classroom</td>
<td>.03</td>
<td>.02</td>
</tr>
</tbody>
</table>

* p < .001.
This study had a few limitations. First of all there were methodological questions about the use of 'how many'-tasks in order to assess the cardinality principle (Le Corre, Van de Walle, Brannon, & Carey, 2006). It has been reported that children simply order to assess the cardinality principle (Le Corre, Van de Walle, Brannon, & Carey, 2006). It has been reported that children simply methodological questions about the use of 'how many'-tasks in order to assess the cardinality principle as such (Freeman et al., 2000; Sophian & Kailihiwa, 1998). Similar findings have been found in research in the order-relevance principle (Cowan, Dowker, Christakis, & Bailey, 1996). Alternative methods like observing if a puppet counts right or not had been formulated as alternatives for checking if children really understood the counting principles (Briars & Siegler, 1984; Le Fevre et al., 2006). Furthermore, the counting tasks were administered in the same order for all children. Although the administration time of these items was rather short, this could possibly have led to fatigue effects. In addition we could not explain why children in our study did not follow the described order in mastering the counting principles because the items of the TEDI-MATH (Grégoire et al., 2004) are too limited in number and time to draw conclusions. Furthermore, a large proportion of the variance remains unexplained. A lot of possible powerful predictors besides the counting abilities were not taken into account in this research. Increasing evidence has been found for an important role of logic thinking abilities (e.g., Stock et al., 2007), numeracy (e.g., Desoete et al., 2004; Roussele & Noël, 2008) and executive functions (e.g., Mazzocco & Kover, 2007; Van der Sluis, de Jong, & van der Leij, 2007) in the development of later arithmetic abilities. Fayol, Barrouillet and Marinthe (1998) found neuro-psychological tests to have good predictive value for performances on arithmetic tests in 5- to 6-year-old children. All of these indicators were not investigated in this research. Finally, context variables such as home and school environment, learning packages and parental involvement (e.g. Reusser, 2000) should be included as stated.

Yet the large group of children that was assessed in this study strengthens the generalizability of the results. In conclusion we can say that less than half of all children in Flanders mastered the three essential counting principles by the end of kindergarten. Moreover, our results revealed that mastery of the counting principles in kindergarten was predictive for scores on arithmetic tests one year later in first grade. Yet there were important differences between schools. Taken into account the large differences in luggage in counting skills children took with them when starting basic schooling and the fact that scores on counting tasks were good predictors for later arithmetic abilities, it is important that teachers in first grade have sufficient attention for the instruction of counting skills.

References


